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A HETEROGENEOUS TIME-STEP ATTRITION
ALTERNATIVE FOR THE MOSCOW
LOW RESOLUTION COMBAT MODEL

by

John C. Stratis

September 1989

Thesis Advisor: Samuel H. Parry

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A Heterogeneous Time-Step Attrition Alternative for the MOSCOW Low Resolution Combat Model

by

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Captain, United States Army
B.S., United States Military Academy, 1980

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
Sept 1989

ABSTRACT

The Method of Screening Operational Concepts of Warfare Model (MOSCOW) is a low resolution analytic tool designed by the RAND Corporation to assist decisionmakers in comparing the performance of alternative warfighting doctrines. Recent analysis of this model suggest its current battle attrition mechanism places unreasonable conceptual limits on the model's usefulness. This thesis considers an alternative way to compute battle attrition which does not suffer from these difficulties.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. BACKGROUND

The U.S. Army currently uses several different computer simulations to determine the effects of new weapons and different strategies on its ability to both wage and win war. By proper use of these simulations, decision makers can discover how to best employ men and equipment. These models vary from those which focus on the characteristics and use of individual weapons to those which concentrate on the characteristics of many weapon systems that are combined or aggregated to form the basic component of a model s structure.

Basically, combat simulations or combat models are divided into two different categories: high resolution and low resolution. High resolution models usually define model structures that represent exactly one piece of equipment such as a tank or artillery howitzer. As such, they concentrate on demonstrating the tactics of weapons employment and allow for significant user input in controlling the flow of battle. Models that represent aggregated forces are in the second category; low resolution models. Typically, these models provide insight to issues such as where large units of men and equipment should be positioned to maximize their contribution to campaign winning. Beyond this, low resolution models also consider logistical factors such as resupply and maintenance of equipment in order to better understand how these factors can influence campaign outcome. Output from these models address issues on a strategic level such as, "How can a commander best use his assets to accomplish the mission?"

Philip J. Romero of the Rand Corporation developed a new low resolution model called MOSCOW (Method of Screening Concepts of Operational Warfare) [Ref.1]. MOSCOW's purpose is to provide a quantitative model that allows users to compare different concepts in waging war. These warfighting concepts, more commonly referred to as doctrines, are those methods that high level leaders have decided to use in waging war (i.e. Mobile Defense as opposed to a Static Defense). The definition of the current Army doctrine can be found in an unclassified Department of the Army Field Manual [Ref. 2].

In developing his model, Philip Romero intended to make the model easy to use. With this purpose in mind, MOSCOW was written on a Lotus spreadsheet (compatible with personal computers) consisting of 1040 x 57 cells and generally takes on the order of seconds to recalculate. The model is capable of aggregating force sizes at the theater level. By calculation using various inputs, MOSCOW produces an output estimating the required friendly force size needed to produce a desired attrition upon a known enemy force.

Past evaluations of MOSCOW suggest that its method for computing aggregate combat attrition performs poorly in certain situations [Ref. 3]. This thesis compares MOSCOW's current attrition mechanism with a possible alternative.

B. PROBLEM DEFINITION

Previous analysis of MOSCOW demonstrated that a fundamental limitation in the model is that it relies solely on the Lanchestrian square law formulation for calculating combat attrition losses. Detailed explanations of Lanchestrian formulations and the use of the square law for attritions can be found in numerous sources [Ref. 4]. This results in two conceptual limitations on the representation of battle processes in MOSCOW [Ref. 3: p. 7]:

1. The proportion of direct and indirect fire systems as a fraction of the total force remains constant over the entire battle.
2. The rate at which targets become available to indirect fire systems is constant for the duration of the battle.

These limitations are arguably unreasonable.

The first limitation is unreasonable simply because it may not necessarily be true. It is natural to expect that direct fire systems (e.g. tanks and infantry fighting vehicles) are killed at a proportionally higher rate than indirect fire systems. Furthermore, it is easy to conceive of concepts of warfighting which rely predominantly on direct or indirect fire as a means of attriting the enemy.

Such concepts would surely not be capable of maintaining a constant proportion of direct and indirect fire weapon systems throughout the duration of a battle or several battles.

The second limitation is unreasonable because it is hard, if not impossible to estimate the constant number of targets available to indirect fire systems. The actual number is in fact a function of scenario, intelligence on enemy positions, availability of target acquisition systems and the current point in the battle itself. It is not known how to "average" or even obtain estimates for the number of targets engaged in each indirect fire engagement.

Thus, the problem confronted by this thesis is to develop an alternative attrition mechanism for the MOSCOW model which eliminates these conceptual limitations.

C. SOLUTION APPROACH

This paper proposes to use a heterogeneous Lanchester attrition mechanism as an alternative to MOSCOW's current square law formulation.

II. MOSCOW'S METHODOLOGY

A. THE BASIC MODEL IN MOSCOW

MOSCOW was designed to meet a need for analysis that existing models were unable to provide.

> "There was a need for a broad, quantitative model that could rapidly provide appraisals of a wide variety of concepts with modest data requirements and a consequent low level of resolution." [Ref. 1: p. x]

Most combat simulation models use killer-victim scoreboards to display the results of a battle due to fixed inputs (fixed inputs are those parameters defined as starting strengths, lethalities etc.). A killer-victim scoreboard displays the outcome of a battle in terms of killer (those vehicles that caused a casualty) and victim (vehicles that were destroyed). Analysis of these results provides analysts with a view of the battle and therefore allows a determination of which systems were most effective. MOSCOW, on the other hand, uses a specified outcome (desired enemy attrition and maximum penetration limit) and estimates the number of friendly forces needed to produce this result.

A model with an ability to determine the force levels needed to achieve a desired outcome is valuable to military commanders. By using MOSCOW, commanders can gain insight on the amount of forces needed to achieve victory. By comparing these results for different scenarios, commanders can measure which force designs may work best in particular scenarios.

> "It [MOSCOW] emphasizes those policy variables that especially pertain to the operational level of war, i.e., those choices within the purview of corps, army group, or theater commanders." [Ref. 1: p. 121]

MOSCOW's internal composition is complex. Inputs consist of over 350 variables within the Lotus spreadsheet. The model is essentially a set of simultaneous equations, and therefore successive recalculations are required for the model to converge to a solution.

Given the starting conditions of any engagement, MOSCOW uses the battlefield geometry (a bounded area both front and rear, with a border between opposing forces) and other initial force parameters such as the enemy rate of advance and the friendly rate of fire to calculate the campaign kill rate with units of number of kills per day per unit. The other rate determined within MOSCOW is called the required kill rate (Equation 1). Simply stated, the required kill rate is the rate at which the friendly force must kill the enemy force to achieve mission success. This rate would have units of the number of units that have to be killed per day. In a given scenario, the friendly force size required to produce the desired battle outcome is calculated by dividing the required kill rate by the campaign kill rate. The following simplified example of MOSCOW's process of calculation gives a flavor of the model's underlying structure and complexity.

$$(\#\text{enemy killed})/(\text{Time available})=\text{Required kill rate} \qquad (1)$$

The commander of the friendly forces desires to destroy 10 enemy divisions within an allowable penetration of 40 kilometers (the friendly rear border). If enemy forces move at a rate of 8 kilometers per day, then it would take them 5 days to reach the rear border. In other words, the friendly forces must kill 10 enemy divisions in 5 days or less.

$$(40\ \text{km}) / (8\ \text{km/day}) = 5\ \text{days} = \text{Time available}$$

$$(10\ \text{enemy divisions}) / (5\ \text{days}) = 2\ \text{div./ day}$$

If the friendly force must mass 1.5 friendly units for each enemy division it faced in an attack, and each friendly unit is capable of destroying .2 enemy divisions per attack, with each attack lasting for two days then the campaign kill rate (Equation 2) of the friendly force would be:

$$(\#\text{kills/atk}) / (\#\text{units/atk})\text{x}(\#\text{days/atk}) = \text{Campaign kill rate}$$

$$(.2\ \text{kills/attack})/(1.5\ \text{units/attack})x(2\ \text{days}/attack) = 0.066\ kills/day/unit \qquad (2)$$

Dividing the required kill rate by the campaign kill rate gives the resulting number of friendly forces needed for mission accomplishment (Equation 3). For more information on the above simple example, see [Ref. 1: pp.160-163].

$$\text{Required kill rate/Campaign kill rate} = \text{\# of friendly units} \qquad (3)$$

(2 div. /day) / (.066 kills/day/unit) = 30 units

To calculate the battle attrition, the battle calculus within MOSCOW comprises the following seven elements:

1. The engagement initiation criterion which is the tactical attacker and defender combat power ratio (combat power is calculated as the number of vehicles times the square root of the lethality coefficient). The units of the lethality coefficient are dimensionless (i.e. lethality has been defined in MOSCOW [Ref. 1] as the portion of enemy vehicles killed per hour). The starting strengths for both sides is also needed.

$$CombatPower = (\text{ \# of vehicles }) \times \sqrt{lethality} \qquad (4)$$

2. The hardness of the vehicles which accounts for each vehicle's vulnerability to destruction.

3. The availability of vehicles depending upon their movements (stationary or moving) and acquisition probabilities.

4. The lethality coefficient of the vehicles which incorporates the enemy hardness and availability. A thesis written by Mark Hanson [Ref. 4], addresses 30 MOSCOW coefficients and usable ranges for their values.

5. The engagement termination criterion (attrition imposed on or suffered by the defender) as an input by the attacker.

6. The engagement duration and vehicle attrition.

7. Blue's consumption during the engagement of additional commodities such as petrolium, oil, lubricants (POL), ammunition etc.

Element 6 is calculated by combining elements 1, 4 and 5. A form of the square law Lanchester equation is then used (Equation 5) to calculate the time needed for the battle. [Ref. 1: pp. 357-358, 370-374]

$$t = \ln \frac{\dfrac{s\sqrt{r}\,R - \sqrt{bB^2 - (1 - s^2)rR^2}}{R\sqrt{r}\ - B\sqrt{b}}}{\sqrt{rb}} \tag{5}$$

Where,

R is the initial size of the enemy force

B is the initial size of the friendly force

r is the lethality of the enemy force

b is the lethality of the friendly force

t is the time in days

s is the percentage of enemy that survive

Once the engagement duration has been calculated, the square law Lanchesterian equation (Equation 6) is used (as opposed to the inverted form of the Lanchester equation used to calculate the time of battle) to calculate vehicle attritions.

$$R(t) = \frac{\left(R - \dfrac{\sqrt{b}}{\sqrt{r}}\,B\right)e^{\sqrt{\mathrm{rb}}\,t} + \left(R + \dfrac{\sqrt{b}}{\sqrt{r}}\,B\right)e^{-\sqrt{\mathrm{rb}}\,t}}{2} \tag{6}$$

Where,

R is the initial size of the enemy force

B is the initial size of the friendly force

r is the lethality of the enemy force

b is the lethality of the friendly force

t is the time in days

R(t) is the surviving strength of the enemy force

B. NECESSITY FOR SPREADSHEET RECALCULATION

MOSCOW, in order to attain the final value of the campaign kill rate, must recalculate its system of simultaneous equations. Philip Romero (MOSCOW's author) states clearly why the need for recalculations exists, and that usually within 12-15 iterations the program values will be close enough to be considered to have converged [Ref. 5: p. 15]. Since MOSCOW uses a system of simultaneous equations, they are dependent upon each other. The constantly changing values (depending on the current battle that is ongoing within MOSCOW) will eventually converge. Listed below are the three main reasons MOSCOW must be recalculated to produce this convergence.

First, vehicles are assumed to be uniformly distributed throughout the battlefield. The distance each unit must move until the desired combat power ratio is attained (Equation 4) is, therefore, inversely related to the density of the units in the zone. In other words, if there is a high density of vehicles in the zone then the movement time is low and vice versa. The density of the vehicles is a function of the output of the model, therefore, the model's output depends upon itself. Note, however, that an increase in the number of units on the battlefield will result in an increase in density which decreases the movement time (to accumulate combat power) which in turn decreases the number of units required (in the zone) and therefore, the system of relationships is logically convergent. [Ref. 1: p.156]

Secondly, air support and the other supporting forces (such as Corps artillery units) are responsible for imposing a certain average amount of delay per day. Some of these supporting assets (such as air support interdiction) attrit the enemy over the course of the entire campaign (a campaign could possibly consist of several battles). Therefore, in longer campaigns, the average delay per day of a fixed asset will be lower than that of a shorter campaign. Thus, an increase in campaign length will lower the average delay per day which in turn will cause the enemy forces to have a higher advance rate which will lower the campaign length and therefore, again, the system of relationships is convergent. [Ref. 1: p. 160]

Finally, the distance between opposing forces will be smaller as the battle progresses. The closer the forces, the more lethal the attacker becomes. As the attacker becomes more lethal, the time needed to impose a specified amount of attrition upon the defender will be lowered. Therefore, the distance between the opposing forces can be larger which in turn will lower the attackers lethality which will in turn increase the time of the battle and, again, these relationships are logically convergent. [Ref. 1: p. 176]

The three reasons listed above explain the need for recalculation of the MOSCOW spreadsheet. Because of this, it is not possible to "see" the results as the battle progresses but only to "see" the starting conditions and final product. The reliance of MOSCOW upon the Lanchester square law for attrition, combined with a dependence of having simultaneous equations eventually converge, is an area for further improvement.

C. PROBLEM AREAS

Although the basic structure of MOSCOW is built upon the need for a model that can output data based upon a specified outcome, it still lacks the ability to properly handle the following four areas.

As previously mentioned, MOSCOW assumes that the ratio of the number of indirect weapon systems to the total force size remains constant throughout the engagement. If we are to believe this assumption, then any attrition delivered to direct fire forces will have proportionally the same effect on indirect fire forces. Common sense tells us that in any tank heavy battle, the direct systems will suffer a much larger attrition rate than the indirect systems simply because the direct systems (tanks) are engaged in a close combat battle while their supporting indirect elements (artillery) are some distance to the rear.

Additionally, once the initial forces within MOSCOW have been aggregated, all subsequent units must be of the same composition. Independent units such as a separate artillery brigade or an armored brigade could not be included in the simulation since their compositions are vastly different than a standard armored or mechanized infantry division (e.g. an artillery brigade consists of 244 howitzers and 0 tanks).

During any engagement, the attacker continues to assault the defender until one of two conditions are met. First, if the attacker successfully achieves a desired level of attrition on the defending force, the attack ceases. Second, if the attacker is annihilated, then the attack again ceases. Common sense would normally prevail in the second case and the attacking commander would call off the attack prior to his unit's destruction.

Finally, allocation of indirect fires is simply omitted. A variable is needed to determine the percent of indirect fire that should be placed upon the enemy's direct and indirect fire weapon systems.

In attempting to resolve the limitations within the MOSCOW model (and the ensuing problems listed above), a heterogeneous Lanchester attrition equation is proposed that separately accounts for the effects due to direct and indirect fire systems.

III. DEVELOPMENT OF THE HETEROGENEOUS MODEL

A. MODEL APPROACH

The heterogeneous model was developed to show a different approach to attrition analysis. By separating the effects of direct and indirect fire, individual contributions of each system can be further analyzed.

MOSCOW uses a deterministic approach involving a system of simultaneous equations because its attrition equation (square law) has a closed form solution. The heterogeneous model will also use a deterministic approach, but a closed form solution is not generally known when attrition equations involve the use of both the square and linear law. As a result, the heterogeneous model will use a time-step algorithm for calculating combat attrition. Appendix A contains the actual program code with detailed variable descriptions. A listing of the inputs used with the program can be found in Appendix B. Further information on the Lanchestrian theory of attrition can be found in research by Taylor [Ref. 6].

B. MODEL ASSUMPTIONS

The heterogeneous model separates the effects of direct and indirect weapon systems. Since the effects of these systems will be calculated separately, every engagement will, therefore, have four weapon systems available for attrition purposes. These weapon systems are listed in Table 1.

TABLE 1. WEAPON SYSTEMS AVAILABLE IN AN ENGAGEMENT

	Force	Type of vehicle
1.	Blue	direct
2.	Blue	indirect
3.	Red	direct
4.	Red	indirect

"Blue" represents friendly forces and "Red" refers to the enemy force.

C. ATTRITION MECHANSIMS

Table 2 lists the weapon systems attrition relationships which are possible in each engagement in the proposed model. Significantly, there are no Firer-Target relationsnips for direct fire systems to engage indirect fire targets. The elimination of these relationships from the model is based on the assumption that this type of attrition makes a small contribution to total battlefield attrition. This is because current indirect fire weapons are doctrinely positioned to preclude the possibility of attack from direct fire weapons. Table 2 lists the Firer-Target relationships contained in the proposed attrition model.

TABLE 2. FIRER VERSUS TARGET ATTRITIONS

	Firer	Target
1.	Blue direct	Red direct
2.	Blue indirect	Red direct
3.	Blue indirect	Red indirect
4.	Red direct	Blue direct
5.	Red indirect	Blue direct
6.	Red indirect	Blue indirect

1. Lanchestrian Linear Law

The linear law is used to model attrition from area fire weapons. This law gives no advantage to concentrating forces on the battlefield. As such, this relationship is oten used to model indirect fire (artillery) weapons [Ref. 7]. Equation 7 is an example of the linear law.

$$dX/dt = -aXY \tag{7}$$

Where,

X represents the number of targets

Y represents the number of firers

dX is the change in the X force

dt is the change in time (time interval)

a is the attrition coefficient

Simply stated, the linear law shows that the resulting attrition to an enemy force is a function of both the number of targets and the number of firers. The attrition coefficient a, is calculated from several inputs. The units of a, are:

(# killed) / (# firers) (# targets) (unit time)

2. Lanchestrian Square Law

Attrition produced by direct fire systems, on the other hand, is often modelled using the Lanchester square law attrition [Ref. 7]. This law models that aspect of war where individual targets can be identified and attacked. This is a model of "aimed fire" and therefore typically used with direct fire weapons. Equation 8 shows the square law formulation.

$$dX/dt = -bY \tag{8}$$

Where,

Y represents the number of firers

dX is the change in the X force

dt is the change in time (time interval)

b is the attrition coefficient

The units of the attrition coefficient b, are:

(# killed) / (# firers) (unit time)

3. Vehicle Attrition Calculation

Using the information from Table 2, Figure 1 illustrates the attrition that will occur given an engagement. Note that the direct fire systems of both forces are attrited by the opposing direct and indirect fire systems, and that indirect fire systems are only attrited by indirect fire systems.

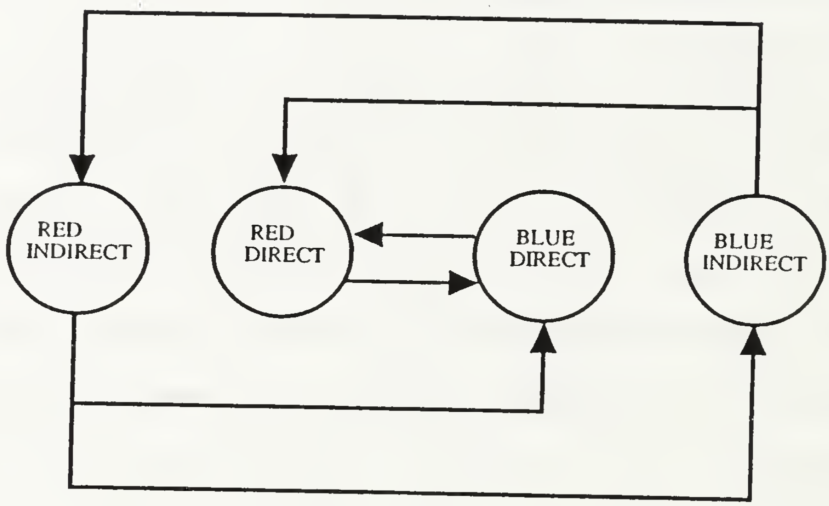

Figure 1. Attrition by different weapon systems

With Blue as the X force, and Red as the Y force, Table 3 shows how the linear law and the square law can be applied to describe the attrition casualties produced by each type of system. Subscripts indicate different attrition coefficients. Explanations of the variables used to calculate the attrition coefficients are in Appendix C.

TABLE 3. APPLICABLE ATTRITION EQUATIONS

Firer	Target	Attrition Equation
Blue direct	Red direct	$dY_D/dt = -a_1X_D$
Blue indirect	Red direct	$dY_D/dt = -a_2X_IY_D$
Blue indirect	Red indirect	$dY_I/dt = -a_3X_IY_I$
Red direct	Blue direct	$dX_D/dt = -b_1Y_D$
Red indirect	Blue direct	$dX_D/dt = -b_2X_DY_I$
Red indirect	Blue indirect	$dX_I/dt = -b_3X_IY_I$

Figure 1 shows that direct fire attrition for each force results from a combination of direct-direct and indirect-direct fires. By combining the appropriate equations, the attrition relationships for direct fire systems (of both X and Y) are:

For the X force $dX_D/dt = -b_1Y_D - b_2X_DY_I$

and,

For the Y force $dY_D/dt = -a_1X_D - a_2X_IY_D$

In comparison, indirect systems are attrited only by opposing indirect systems (counterbattery battles). The resulting attrition relationships are:

For the X force $dX_I/dt = -b_3X_IY_I$

and,

For the Y force $dY_I/dt = -a_3X_IY_I$

D. DIFFERENCES BETWEEN THE TWO MODELS

Although many similarities exist between the two models (MOSCOW and the heterogeneous model), there are some differences. To understand the development of the heterogeneous model, these differences need to be illuminated.

1. The Changing FLOT (Front Line of Troops)

MOSCOW iterates (recalculates) its simultaneous equations until convergence. At the point of convergence, the average distance between the two forces is calculated using the following equation.

$$\textit{average distance} = \frac{\textit{initial distance} + \textit{final distance}}{2} \tag{9}$$

The resulting answer is then used as the distance between the two forces throughout the engagement (as if they had remained stationary).

The heterogeneous model, on the other hand, calculates the distance between the forces for each time interval (the time-step interval). The speed of the unit multiplied by this time interval computes the distance moved in the time interval. Calculations that determine vehicular speed are shown in the following example.

The vehicular speed of the Blue's indirect forces has an input value of 25 kph. If the commander estimates that at any moment in the battle approximately 2/3 of his force will be firing while the remaining 1/3 will be moving, then the speed calculated for the indirect force is shown in Equation 10.

$$(25 \text{ kph})(1/3) + (0 \text{ kph})(2/3) = 8.33 \text{ kph} \qquad (10)$$

Therefore, the speed of 8.33 kph would be used for Blue's indirect vehicles. The values for the indirect vehicles are from an illustration by Hoffman [Ref. 8: pp. 24-25].

2. Allocation of Fire

Decisions are made by military commanders to dedicate certain portions of their force against a specific portion of the enemy (i.e. 30% of the indirect force may be dedicated to fire only at indirect fire systems while the remaining 70% may be firing only at the direct fire systems). MOSCOW does not have a mechanism to describe this allocation. The heterogeneous model uses an additional variable within attrition coefficient calculations to permit a description of a specific force allocation scenario.

3. Attacker Annihilation

MOSCOW fails to determine a breakpoint at which the attacker will quit the battle should he receive a large amount of attrition. Without this feature, it is possible that the attacker, in an attempt to attrit the enemy, will attack until he is annihilated. This clearly makes no sense. The heterogeneous model uses an input that allows the user to determine a maximum level of attrition he is willing to receive in an attack.

4. Range Degradation

An additional factor used in the heterogeneous model accounts for degradation of weapon performance as a function of range. Bonder, [Ref. 7: pp. 30-32] derived the following equations from his studies on range dependency.

$$dX/dt = -a(r)Y \tag{11}$$

$$dY/dt = -b(r)X \tag{12}$$

In Equations 11 and 12, the value of the function r is derived from the following formula:

$$a(r) = \begin{cases} a_0(1 - r/r_{max})^u & \text{for } 0 \leq r \leq r_{max} \\ 0 & \text{for } r \geq r_{max} \end{cases} \tag{13}$$

Where,

r is the current range of the two opposing systems and

r_{max} is the maximum effective range of the weapon

u is the exponential factor, unique for each weapon system.

Figure 2 shows the effects of different values of u in Equation 13. Tanks, as direct fire weapons, would use values of u close to 1. At short ranges, there would be a high probability of kill, and at longer ranges a linear relationship with a lower probability of kill. Artillery, on the other hand would use a much smaller value of u (close to 0) that would indicate a very small range effect on the probability of kill (range has a small effect for artillery unless the range is either less than the minimum range of the weapon or beyond the maximum range of the system). Probability of kill for both weapon systems reaches zero when the range exceeds the maximum effective range.

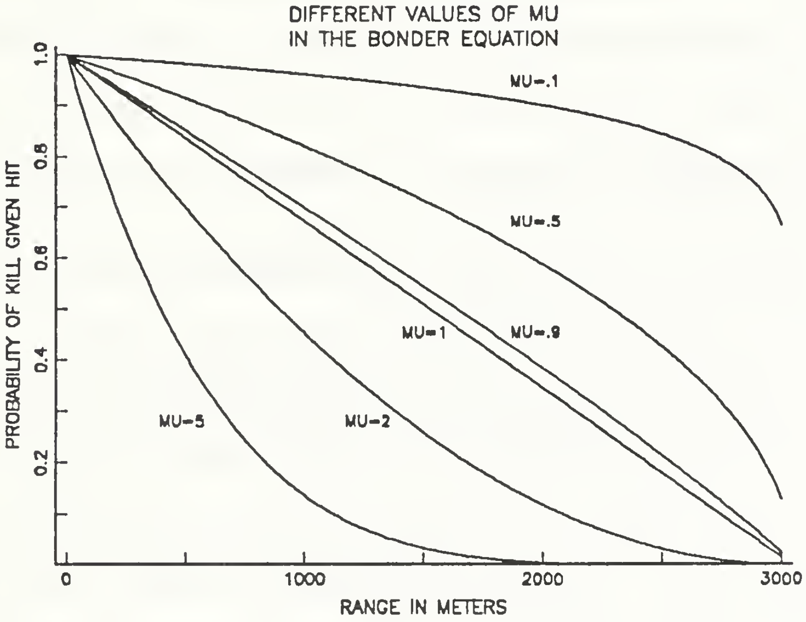

Figure 2. Various values of u for the Bonder range equation

E. BREAKPOINT CRITERION

1. Attacker Breakpoint

As previously mentioned, a primary difference between the heterogeneous model and MOSCOW is that MOSCOW has no breakpoint criterion for the attacker (i.e., the attacker can possibly attack until his annihilation). The criterion used in the heterogeneous model is an input value; i.e. the user inputs a value representing the maximum amount of attrition he is willing to accept while continuing the attack.

2. Defender Breakpoint

Determining the breakpoint criterion for the defender is a different process which involves the following parameters:

1. ATKDES-The attacker's desired attrition upon the defender (e.g. 75%).
2. DEFDES-The defender's tolerable attrition should he be attacked (e.g. 50%).
3. DISENG-The fraction of engagements that the attacker is able to dictate the duration (e.g. 80%). This variable attempts to illuminate a force's ability to "control" the length of the battle.

To illustrate this process, consider the following example: Blue attacks Red and desires to attrit Red by 75%. Blue is also able to dictate 80% of the duration of engagements, while Red desires to lose no more than 50% of his force should Blue attack. MOSCOW (and the heterogeneous model) use the following formula (Equation 14) to determine the defender (Red) breakpoint criterion.

Defender Breakpoint=DEFDES + (DISENG x (ATKDES-DEFDES)) (14)

Using the values from the example above, the resulting defender breakpoint criterion becomes:

Defender Breakpoint = .5 + (.8 x (.75-.5)) = 70% attrition.

The value (70%) indicates that although the attacker (Blue) wishes to attrit Red by 75%, he only "controls" 80 percent of the engagement, and therefore is unable to achieve his desired attrition level. Red, on the other hand, desires to lose no more than 50 percent, but is unable to control the flow of the battle, and is therefore forced to continue defending until he reaches 70 percent attrition, or Blue reaches its breakpoint (whichever comes first).

3. Artillery Breakpoint

For both Red and Blue indirect forces, the breakpoint attrition level is an input value. When this value is reached, all of a force's indirect fire systems are considered to be ineffective and no longer participate in the battle. The remaining indirect systems (of the effective force) then change their allocation of fire and concentrate 100 percent of their fire on the opposing direct fire systems. If a direct fire breakpoint is reached, the direct fire battle ends immediately and indirect fire systems on both sides concentrate 100 percent of their fires on a counterbattery battle (which may last much longer than the direct-direct battle).

F. VALUE OF THE HETEROGENEOUS MODEL

The model was developed for several purposes. First, it can be used as a tool to assist commanders in their decision making process. Commanders are allowed greater flexibility in changing their force structure by determining the possible consequences of their decisions as a result of each weapon system's independent attrition. Additions of non-homogeneous units (i.e. artillery brigades) are not allowed in the current version of the MOSCOW model.

Additionally, the model forecast force augmentation configurations necessary for success given a predicted Red threat scenario. Finally, the heterogeneous simulation may be stopped at any point in the battle for analysis and then modified and continued if desired.

IV. ANALYSIS OF THE HETEROGENEOUS MODEL PERFORMANCE

A. ANALYSIS WITHIN AN ENGAGEMENT

Models are designed to simulate possible outcomes of engagements given certain input conditions. Since these battles have not actually occurred, real data for comparison purposes are not available. Therefore, analysts are left to assess a model's validity by evaluating its performance according to expectations based on informed judgement.

By comparing the results of one model with those of another, model differences can be seen by specifying a common scenario. Analysts can then gauge the degree of "within engagement" difference between the two models representations of battle processes. If these differences are relatively small, then there is arguably no difference between the models and the choice of using one model over another must be made based on other criteria such as computational complexity, ease of use, or other factors.

A comparison of MOSCOW's current attrition process and the heterogeneous attrition model is especially simple using the ratio of the numbers of indirect to direct fire weapon systems. As Hoffman [Ref. 3] shows, the assumptions underlying the current MOSCOW attrition process fix this ratio as a constant value of the systems present at the start of an engagement. Using the heterogeneous model, the value of this ratio typically fluctuates during an engagement. Figure 3 shows a comparison of how this ratio varies for the proposed heterogeneous attrition model and the performance of the current MOSCOW attrition algorithm during a hypothetical engagement.

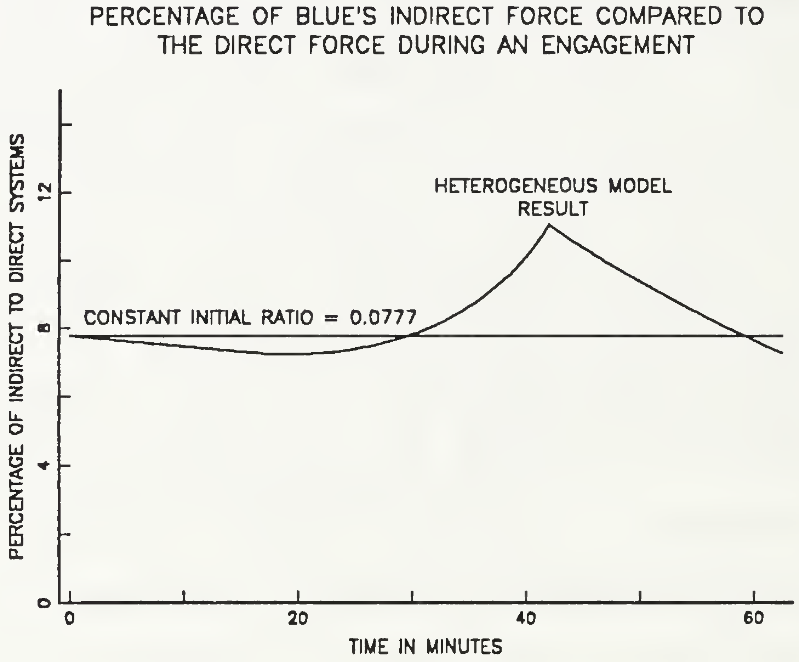

Figure 3. A comparison of the ratio of Blue's indirect to direct weapons for MOSCOW and the heterogeneous model during an engagement.

Model inputs for the initial force levels are shown in Table 4. These inputs were derived from simulating an engagement of two Red (Soviet) divisions opposed by a Blue (U.S.) force. The Blue force is attacking with a four-to-one advantage in total weapon systems.

TABLE 4. FORCE STRUCTURES OF A HYPOTHETICAL ENGAGEMENT.

Weapon system	Number of Vehicles
Red direct fire	1557
Red indirect fire	424
Blue direct fire	6228
Blue indirect fire	484

A statistical measure of the degree to which these ratios differ is given by the R^2 statistic defined by Equation 15 [Ref. 9: p.640]. This is an application of the R^2 statistic common to regression analysis. In this case, R^2 provides a measure of the degree to which the constant ratio result of the current MOSCOW model agrees with the results of the heterogeneous model.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - c)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{15}$$

Where,

$\bar{y}$ is the average value of the sum of the y_i's

y_i is the value of the heterogeneous model for $i=1,2,...$

c is the constant initial force ratio value (0.0777)

In this case, R^2 has a value of -0.2066. Therefore, by this statistical comparison, these two models do not give equivalent representations of combat attrition.

B. BETWEEN ENGAGEMENT ANALYSIS

1. Red Force Allocation Scenarios

Recall that MOSCOW not only assumes a constant ratio of direct to indirect systems within a single engagement, but also uses this same value for all engagements. Therefore, another way to assess model performance is to analyze a model's responses over a range of plausible scenarios to see if results are reasonable (follow logical attrition expectations). For this analysis, five scenarios represent a likely range of force allocation options for the Red force given a fixed number of combat systems. Starting with a basic scenario representing the doctrinal organization of two Soviet maneuver divisions (1849 total weapon systems), these scenarios contain different allocations of direct and indirect fire systems. While the total number of systems is held constant, the differences in allocation (or force mix) are those which could conceivably result from changes in doctrine, constraints imposed by logistics, or prior destruction of forces by enemy action. The intent of these scenarios is to portray not only a doctrinal employment of Red forces, but also a range of force allocations.

There are two obvious allocation options which a Red force commander might face. The first is a situation which produces a shortage in indirect fire (artillery) systems. This might result from enemy action which has attrited these systems and a logistics system which is unable to provide timely replacements. Faced with this circumstance, a commander may decide to allocate additional direct fire (tank) forces. Thus, the doctrinal allocation of weapon systems has been necessarily altered by the realities of combat. Similar reasoning easily constructs situations compelling the second allocation option; a case where indirect fire systems are used to make up a shortage in direct fire means. The resulting force allocation indicates a necessary preponderance of indirect fire assets. This range of options is seen in the force levels for the five scenarios considered in this analysis as seen in Table 5. Scenario 1 is the case where Red is short of indirect fire systems and attempts to compensate by adding direct fire. Scenario 5 is the situation in which additional Red indirect fire weapons redress a shortage of direct fire systems. Scenario 3 is the doctrinal or base case allocation of these systems, while Scenario 2 allocation falls between Scenarios 1 and 3 while Scenario 4's allocation is between Scenarios 3 and 5.

TABLE 5. VEHICLE BREAKDOWN FOR THE FIVE SCENARIOS.

	# of Red Direct systems	# of Red Indirect Systems	Percentage of direct systems to total systems
Scenario 1	1749	100	94.6
Scenario 2	1653	196	89.4
Scenario 3	1557	292	84.2
Scenario 4	1453	396	78.6
Scenario 5	1349	500	73.0

2. Four Cases of Blue Force Allocation

Each Red force listed in Table 5 can be compared against four Blue force allocations. Current military doctrine [Ref. 2] suggests the attacker should have between three to five times the force size of the defender. Therefore, for this analysis the Blue force will initially have three times the number of Red direct fire systems in each scenario (approximately 4-5 U.S. divisions) and represent the base case for Blue. Three other cases are developed for the Blue force by a) adding one additional division (1000 direct vehicles, "pure direct"), b) adding 700 indirect vehicles representing three artillery groups ("pure indirect"), and c) a combination of a and b above ("balanced allocation"). Table 6 represents the twenty combinations of Red and Blue force structures that are used for analysis purposes. The first number in each column represents the number of direct fire systems of the Blue force and the second represents the number of indirect fire systems.

TABLE 6. BLUE FORCE STRUCTURES USED IN EACH SCENARIO

	CASE # 1	2	3	4
Scenario 1	5300/300	6300/300	5300/1000	5800/650
Scenario 2	5000/300	6000/300	5000/1000	5500/650
Scenario 3	4700/300	5700/300	4700/1000	5200/650
Scenario 4	4400/300	5400/300	4400/1000	4900/650
Scenario 5	4100/300	5100/300	4100/1000	4600/650

3. Weapon System Attrition as a Measure of Effectiveness

Obviously, the friendly force commander desires to minimize his casualties while maximizing enemy casualties. The heterogeneous model represents the interaction of four attrition components, namely, the direct and indirect weapons on each side. It is natural to use a comparison of the attrition of these components as measures of effectiveness (MOE) which characterize any given battle. This is because it is logical to assume that attrition beyond a certain point equates to defeat.

4. Analysis

Because the heterogeneous model separates the direct and indirect fire attrition effects, results can be used to depict the individual contribution of each system to enemy attrition. MOSCOW is unable to do this since it cannot change the "fixed" percentage of indirect fire regardless of the Red and Blue force sizes. Figure 4 diplays a typical example of how indirect fire can separately contribute to the enemy's attrition. The chart represents the percentage of Red direct fire casualties caused by Blue direct and indirect weapon systems. Blue's force structure is displayed on the X-axis by the number of direct and indirect fire systems. The numbers above each bar are the actual number of vehicles killed by the specific weapon system. Even though cases 2 and 3 both attrited the Red direct fire systems to the Red breakpoint criterion (70%), the relative contribution of Blue indirect versus direct vehicles is distinctly different for the two cases. Furthermore, a comparison of cases 1 and 3 shows an increase in Red attrition (366 vs. 640) when Blue's indirect vehicles are increased from 300 to 1000 and holding Blue's direct vehicles constant (5300). The contribution of indirect fire is significantly more in case 3 than in case 1. Appendix D is a collection of graphs depicting alternative methods of representing separate attrition effects that occur in an engagement.

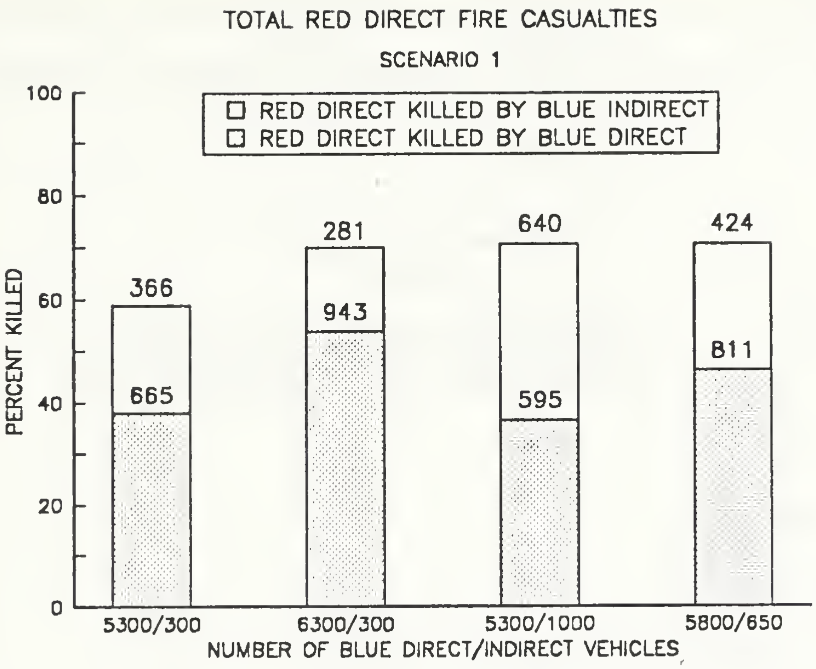

Figure 4. Total Red direct fire casualties, Scenario 1

When the Blue force has a large amount of indirect weapon systems, the counterbattery battle is over quickly and therefore Blue is able to concentrate on attriting Red's direct systems earlier in the engagement. An analysis of Figure 5 indicates that although Blue has caused Red to reach its indirect system breakpoint criterion (70%) in cases 2,3, and 4, the resulting percent attrition suffered by Blue is not the same. In other words, case 3 demonstrates that Blue overpowers the Red indirect force, in turn receives essentially the same number of casualties (although a much smaller percentage) and contributes significantly to Red's direct vehicle attrition (Figure 4).

Because MOSCOW cannot calculate the separate effects caused by indirect fire, this entire analysis is not available with a square law model using a single "generic" weapon system type.

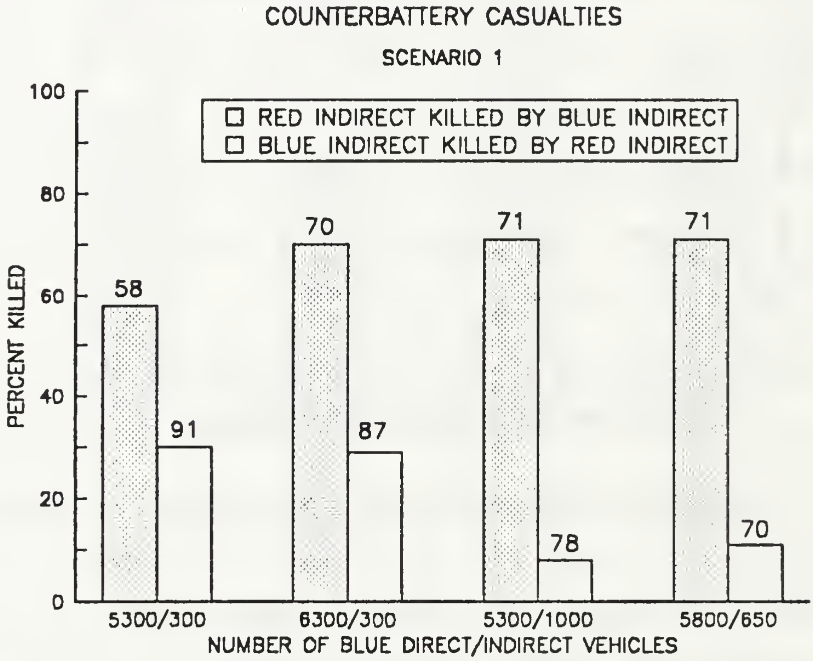

Figure 5. Counterbattery casualties, Scenario 1

The following graphs (Figures 6-10) illustrate the entire range of Red scenarios. Analysis of these graphs shows that the Blue force performance is heavily dependent on the Red force composition. Note the trend displayed on each graph as the Red scenario changes.

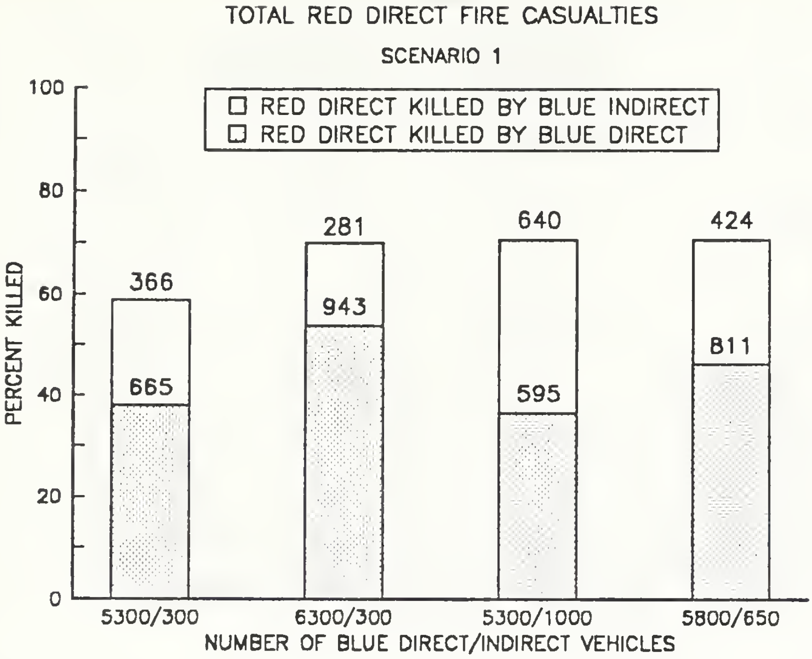

Figure 6. Total Red direct fire casualties, Scenario 1

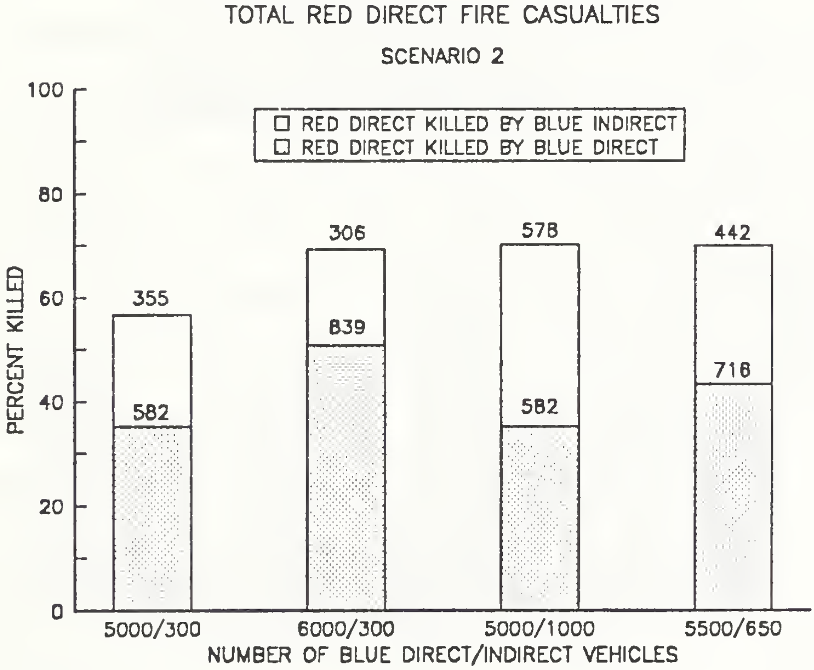

Figure 7. Total Red direct fire casualties, Scenario 2

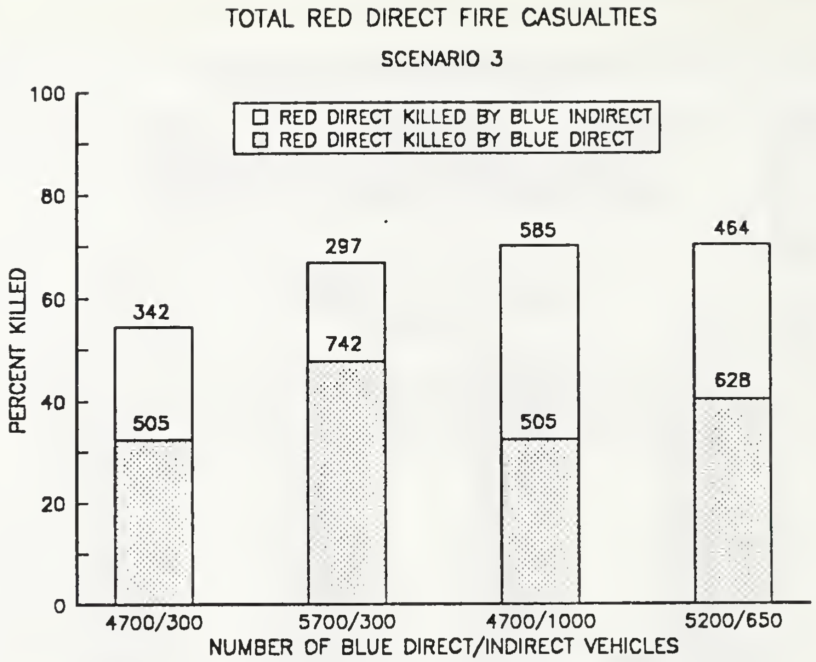

Figure 8. Total Red direct fire casualties, Scenario 3

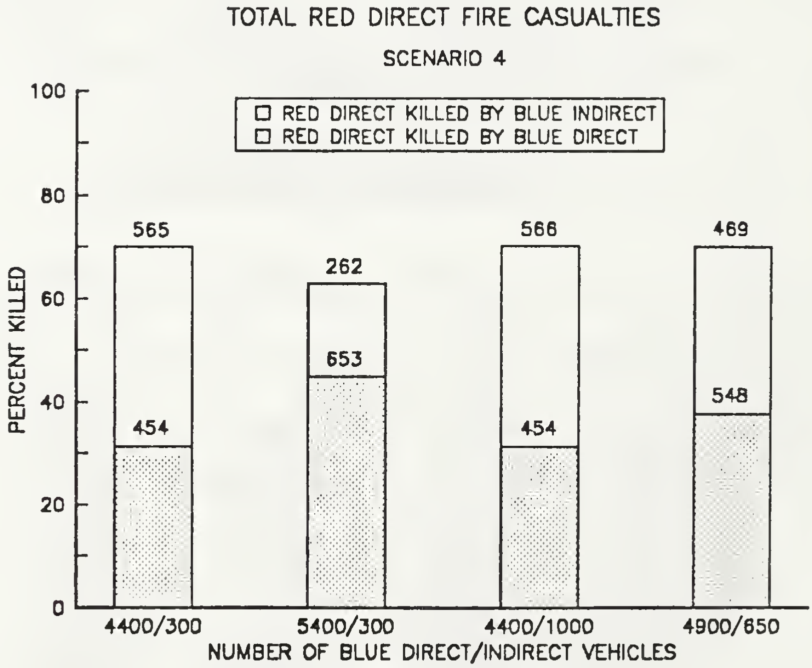

Figure 9. Total Red direct fire casualties, Scenario 4

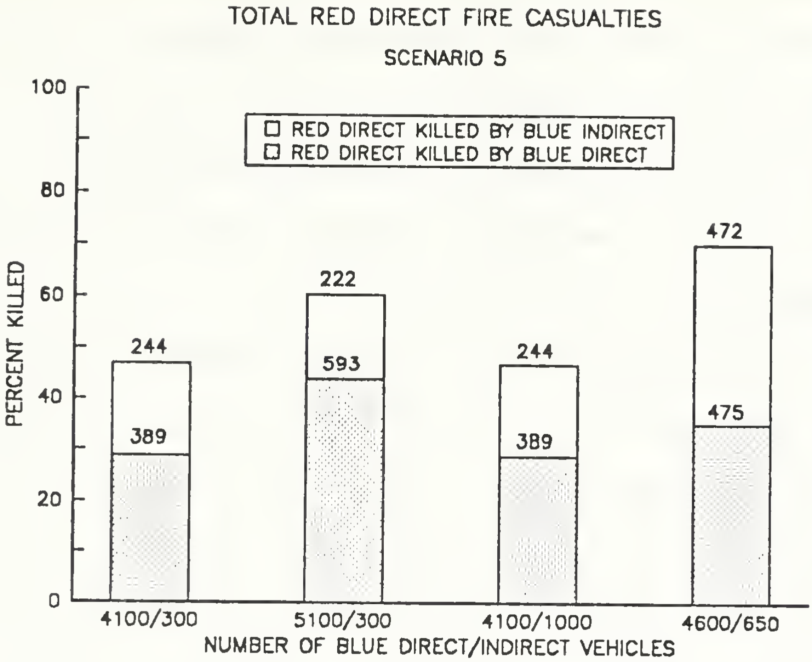

Figure 10. Total Red direct fire casualties, Scenario 5

Three comparisons will be discussed in detail using the five graphs. First, Case 2 demonstrates the importance of augmenting the Blue force with a comparable type attrition weapon as that used by Red enemy force augmentation. This implies that as Red augments with direct fire systems, then Blue should do likewise. In the first three scenarios, Case 2 is able to maintain a 70% direct fire casualty attrition on the Red force. But, when the scenario changes to either scenario 4 or 5, Red's attrition is below its breakpoint criterion (i.e., Blue is no longer able to cause Red to lose 70% of his direct fire systems). This result occurred because Red's composition was increasing in indirect fire systems, the Blue force was augmenting with direct systems (vehicle mismatch) and, therefore, Blue is unable to attrit the Red force as much as in the earlier scenarios. Given the history of mechanized warfare in the twentieth century, this relationship seems reasonable.

The second comparison involves the third Blue case ("pure indirect"). In this case, the Blue force has 1000 indirect systems while the Red force initially has only 100 (Scenario 1). As mentioned earlier, this large weapons ratio (10:1), allows Blue to easily defeat the Red artillery and assist early in the direct attrition battle. But, as the Red force increases its number of indirect vehicles (Scenarios 2-5), the Blue force must allocate more of its weapon systems to the counterbattery war and therefore has fewer systems remaining in which to assist in the direct attrition battle. Finally, in Scenario 5, the Red force has accumulated sufficient indirect systems so that the Blue indirect systems are kept at bay and therefore their contribution to the direct attrition is severely reduced (from 640 to 244).

The third comparison involves the augmented mix version of Blue's force structure (Case 4). Scenario 5 clearly shows that while the attrition of Red decreased for both Case 2 and Case 3, the augmented mix structure of the Blue force was able to cause Red to be attrited to its breakpoint. In fact, Case 4's attrition on the Red force is 70% for all five scenarios. Therefore, as the enemy force increased the ratio of its indirect fire systems, an augmented mixed force did consistently better than either a "pure direct" or "pure indirect" addition. The reason is that as Red's force structure changed, Blue's composition changed to meet the new threat (i.e. a "pure direct" or a "pure indirect" augmentation fared worse than a balanced augmentation of both direct and indirect systems). Case 4's augmentation of indirect systems was able to offset the increase in Red's indirect systems. As more of Blue's indirect systems were used to fight the counterbattery war (resulting from Red's increase in indirect weapons), the resulting decrease of Blue's indirect effort of attriting the direct forces was offset by the augmentation of direct systems.

C. CONCLUSION

The importance of a heterogeneous allocation and attrition model is that it allows analysis to be conducted both "within" and "between" engagements by using different force structures for the Blue force. The previous graphs demonstrated that although an augmented mix version of Blue's force structure tends to perform well in all scenarios, the "best" Blue structure is heavily dependent upon the composition of the Red (enemy) force (different force mixes for Blue produce different results when facing similar Red forces). MOSCOW, on the other hand, is unable to alter the force structures for different engagements. Therefore, over a range of five different and plausible scenarios, the heterogeneous model demonstrates proper battle dynamics which the MOSCOW model cannot show.

V. CONCLUSIONS AND RECOMMENDATIONS

The development of MOSCOW by Philip Romero was an attempt to fill a void in current combat models. It has been noted, however, that there are problems as the model currently exists [Ref. 3], but it is this author's opinion that as an individual project, Romero did a good job.

A. CONCLUSIONS

The use of certain assumptions within MOSCOW pose limitations in analyzing output and therefore tends to overstate or understate the actual remaining force structures. The reason is simple; MOSCOW offers a trade-off in model complexity for resolution by using a single attrition coefficient. On the other hand, the use of a heterogeneous model, although more complex in its attrition development, appears to provide adequate results and clearly overcomes the conceptual problem of treating indirect fire in a Lanchester square law context.

If MOSCOW is to be useful, then an appropriate model should allow for the introduction of different force mixes to be used in analysis given varying threat compositions. Commanders do not think in terms of generic units but rather in terms of direct and indirect forces available for use in battle.

B. RECOMMENDATIONS

A heterogeneous Lanchester attrition representation capable of separately calculating the effects of both direct and indirect fire within the structure of MOSCOW would allow users to have more flexibility in developing their force compositions for analytical purposes.

The development of a heterogeneous model could be used as a tool to determine the type(s) of Blue force structures that are best suited to warfighting given predicted scenarios for Red forces.

C. POSSIBLE EMBELLISHMENTS

Refining the internal programming of MOSCOW (or defining the specific approach) necessary to allow the insertion of a heterogeneous attrition representation is a broad area for future research. The intent of subsequent research would be to restructure the current version of MOSCOW so that it may maintain its concept of warfighting capability but have the flexibility to handle heterogeneous units as opposed to generic units. A further study involving the effects caused by the attrition of indirect fire systems by direct systems may be useful in the evaluation of future warfighting concepts.

The heterogeneous model, as it currently exists, is a descriptive model; i.e., it describes the effects of certain input force structures. The development of a prescriptive model which informs commanders as to the type of forces best suited to war winning would certainly assist them in their decisions.

Finally, the development of an optimizing program (possibly involving a search for the "optimal" path) to determine the composition of the "best" Blue structure given a) possible choices of Red threats, or b) a defined Red threat, would be useful to commanders. This method could involve network methodology that maximizes the minimum paths of one network while minimizing the maximum flows of another network.

APPENDIX A. PROGRAM CODE FOR THE HETEROGENEOUS MODEL

The heterogeneous model was programmed in APL due to its ease in handling vectors and matrices. The actual code of the program is listed on the following pages. Once the initial inputs have been established (and placed within the program code), the model is simple to use. Following a prompt, the user inputs his desired number of direct and indirect weapon systems. Calculations are then made such as the number of vehicles able to fire, the attrition caused by each weapon system type and the distance moved by these systems. Weapon systems are allowed to shoot and move in every time interval.

The percentage remaining (by weapon system) is then output. The program updates all variables (e.g. number of direct fire systems remaining) and conducts a check with the breakpoint criterion as discussed in Chapter 3. If no breakpoints are reached, the model then calculates the attrition that will occur in the next time interval until a system's breakpoint criterion has been reached.

At this point in the engagement (based upon the input data), a decision is made as to battle success or failure and outputs the results. Model run time is on the order of seconds. A sample output of the model is included at the end of the program listing.

The following list is a description of the variables that are used in the APL program developed for analysis in this thesis.

AIJ.......Attrition coefficient for Red firer.
AJI.......Attrition coefficient for Blue firer.
ALLOIJ....Allocation of fire priority for Red firer.
ALLOJI....Allocation of fire priority for Blue firer.
ATTR......Actual attrition to be imposed on Red force.
BD........Cumulative vector of Blue direct casualties.
BHTMN3....Blue vehicles hits per minute calculation #3.
BI........Cumulative vector of Blue indirect casualties.
BIRATI....Cumulative vector of Blue's ratio of indirect to direct fire systems.
BLUATT....Blue will stop the counterattack if Blue receives this much attrition.
BLUDES....What Blue desires to attrit on the Red force.
BLUDIR....Percent of Blue direct vehicles surviving.
BLUID1....Firing rate for Blue firers, calculation #1.
BLUID2....Firing rate for Blue firers, calculation #2.
BLUID3....Attrition rate for Blue indirect firers calculation #3.
BLUID.....Final attrition rate for Blue indirect firers vs. Blue direct.
BLUII.....Final attrition rate for Blue indirect firers vs. Blue indirect.
BLUIND....Percent of Blue indirect vehicles surviving.
BRANVA....Range variable for Blue vehicles.
CABDRD....Cumulative casualties to Red direct by Blue direct.
CABIRD....Cumulative casualties to Red direct by Blue indirect.
CABIRI....Cumulative casualties to Red indirect by Blue indirect.
CARDBD....Cumulative casualties to Blue direct by Red direct.
CARIBD....Cumulative casualties to Blue direct by Red indirect.
CARIBI....Cumulative casualties to Blue indirect by Red indirect.
CASBD.....Blue force direct fire system casualties.
CASBI.....Blue force indirect fire system casualties.
CASD......Total direct fire system casualties.
CASI......Total indirect fire system casualties.
CASRD.....Red force direct fire system casualties.
CASRI.....Red force indirect fire system casualties.
CBD.......Flag for Blue direct breakpoint.
CBI.......Flag for Blue indirect breakpoint.
COUNT.....Counter variable for increasing force sizes.

CRD.......Flag for Red direct breakpoint.
CRDBD.....Flag for Red and Blue direct breakpoint.
CRDBI.....Flag for Red direct and Blue indirect breakpoint.
CRI.......Flag for Red indirect breakpoint.
CRIBD.....Flag for Red indirect and Blue direct breakpoint.
CRIBI.....Flag for Red and Blue indirect breakpoint.
CUMRAN....Vector of ranges each time interval.
CUMTIM....Vector of all times used in simulation.
CURRAN....Current range between opposing forces.
CURBR.....Current range for Blue firers, Red targets.
CURRB.....Current range for Red firers, Blue targets.
C3ERR.....C3 Error (degradation).
DELTAT....Time interval (minutes) for time step.
DISENG....The fraction of engagements in which Blue is presumed to be able to dictate the duration of the battle.
DSMVBD....Distance moved by Blue direct vehicles.
DSMVBI....Distance moved by Blue indirect vehicles.
DSMVRD....Distance moved by Red direct vehicles.
DSMVRI....Distance moved by Red indirect vehicles.
FIRRAM....Average firing rate per system when moving.
FIRRAS....Average firing rate per system when stationary.
HITRMM....Probability of hit per round, firer moving, target moving.
HITRMS....Probability of hit per round, firer moving, target stationary.
HITRSM....Probability of hit per round, firer stationary target moving.
HITRSS....Probability of hit per round, firer stationary target stationary.
HTMIN1....Hit per minute calculation #1.
HTMIN2....Hit per minute calculation #2.
I.........Counter used in do-loop for row variable.
INITLD....Initial number of direct fire vehicles.
INITLI....Initial number of indirect fire vehicles.
J.........Counter used in do-loop for column variable.
KILLHB....Probability of kill given hit for Blue firer.
KILLHR....Probability of kill given hit for Red firer.
KPRBI.....Kill per round probability for Blue indirect fire.
KPRRI.....Kill per round probability for Red indirect fire.
LOBDRD....Casualties to Red direct by Blue direct.
LOBIRD....Casualties to Red direct by Blue indirect.
LOBIRI....Casualties to Red indirect by Blue indirect.
LORDBD....Casualties to Blue direct by Red direct.
LORIBD....Casualties to Blue direct by Red indirect.
LORIBI....Casualties to Blue indirect by Red indirect.
MAXEFF....Maximum effective range of weapon systems.
MISCB.....Blue miscellaneous multipliers (degradation).
MISCLE....Miscellaneous multiplier (degradation) for lethality.
MISCR.....Red miscellaneous multipliers (degradation).

MISCVU....Miscellaneous multiplier (degradation) for vulnerability.
MU........Weapon accuracy parameter used as an exponent.
NUMVED....Number of direct fire vehicles.
NUMVEI....Number of indirect fire vehicles.
PERMM.....Percent of engagement firer moving, target stationary.
PERMS.....Percent of engagement firer moving, target stationary.
PERSM.....Percent of engagement firer stationary, target moving.
PERSS.....Percent of engagement firer stationary, target stationary.
RANGE.....Initial starting range in meters.
RD........Cumulative vector Red direct casualties.
REDDES....What Red desires to be attrited by a Blue attack.
REDDIR....Percent of Red direct vehicles surviving.
REDID1....Firing rate for Red firers, calculation #1.
REDID2....Firing rate for Red firers. calculation #2.
REDID3....Attrition rate for Red indirect firers calculation #3.
REDID.....Final attrition rate for Red indirect firers vs. Blue direct.
REDII.....Final attrition rate for Red indirect firers vs. Blue indirect.
REDIND....Percent of Red indirect vehicles surviving.
RHTMN3....Red firers hits per minute calculation #3.
RI........Cumulative vector of Red indirect casualties.
RRANVA....Range multiplier for Red vehicles.
TIME......Current time in simulation.
TOTAL.....Dummy variable used for output calculations.
VEFIBR....Percent of vehicles firing. Blue vs. Red.
VEFIRB....Percent of vehicles firing. Red vs. Blue.
VEHSPD....Vehicle speed in meters per minute.

THE HETEROGENEOUS MODEL CODE IS LISTED BELOW.

```
⍝INITIALIZE VARIABLES
⍝
⍝TIME INTERVAL ΔT IN MINUTES
  DELTAT←1.0
 COUNT←0
 RD←0
 BD←0
 RI←0
 BI←0
⍝
⍝INITIALIZE COUNTER VARIABLES
 CBD←0
 CRD←0
 CRI←0
 CBI←0
 CRDBD←0
 CRIBI←0
 CRDBI←0
 CRIBD←0
⍝MU IS THE WEAPON ACCURACY PARAMETER
 MU← 2 2 ⍴ 0.9 0.9 0 0
⍝
⍝KILL PER HIT CALCULATIONS
 KILLHR← 2 2 ⍴ 0.65 0 0.1 0.4
 KILLHB← 2 2 ⍴ 0.65 0.1 0 0.4
⍝
LOOP80:
⍝ALLOCATION OF FIRE VARIABLES
 ALLOIJ← 2 2 ⍴ 1 0 0.6 0.4
 ALLOJI← 2 2 ⍴ 1 0.7 0 0.3
⍝
⍝NUMBER OF VEHICLES,DIRECT
 NUMVED← 1557 2976
 NUMVEI← 424 243
⍝
⍝INPUT DESIRED NUMBER OF DIRECT AND INDIRECT VEHICLES
⍝FOR THE BLUE FORCE
 'INPUT DESIRED NUMBER OF DIRECT VEHICLES'
 NUMVED[2]←⎕
 'INPUT DESIRED NUMBER OF INDIRECT VEHICLES'
 NUMVEI[2]←⎕
 BIRATI←NUMVEI[2]÷NUMVED[2]
⍝
 →(COUNT=0)⍴LOOP81
 NUMVEI[2]←NUMVEI[2]+(COUNT×10)
LOOP81:
⍝MAINTAIN VARIABLE FOR INITIAL FORCE SIZES
 INITLD←NUMVED
 INITLI←NUMVEI
⍝
⍝CUMULATIVE LOSSES BY FIRER PER TARGET
 LORDBD←0
 LORIBD←0
 LORIBI←0
 LOBDRD←0
 LOBIRD←0
 LOBIRI←0
⍝
⍝KILL PER ROUND FOR INDIRECT FIRE
 KPRRI← 0.00002 0.00006
 KPRBI← 0.00002 0.00012
⍝
⍝MISCELLANEOUS LETHALITY AND VULNERABILITY MULTIPLIERS
⍝
 MISCLE← 2 2 ⍴ 1 1 1 1
 MISCVU← 2 2 ⍴ 1 1 1 1
⍝
⍝INITIAL STARTING RANGE IN METERS
```

```
[69]  RANGE← 2 2 ρ 5000 10000 10000 15000
[70]  CURRAN←RANGE
[71] ⍝
[72] ⍝CUMULATIVE RANGE VECTOR
[73]  CUMRAN←CURRAN[1;1]
[74] ⍝
[75] ⍝INITIAL VEHICLE SPEED IN METERS PER MINUTE
[76]  VEHSPD← 2 2 ρ 60 120 20 50
[77] ⍝
[78] ⍝CURRENT TIME OF SIMULATION
[79]  TIME←0
[80] ⍝
[81] ⍝VECTOR OF TIME INTERVALS
[82]  CUMTIM←TIME
[83] ⍝
[84] ⍝STOPPING CRITERIA
[85] ⍝
[86] ⍝BLUE'S DESIRED ATTRITION ON RED FORCES
[87]  BLUDES←0.75
[88] ⍝
[89] ⍝WHAT RED DESIRE'S TO LOSE SHOULD BLUE ATTACK
[90]  REDDES←0.5
[91] ⍝
[92] ⍝THE FRACTION OF ENGAGEMENTS IN WHICH BLUE IS PRESUMED TO
[93] ⍝BE ABLE TO DICTATE THE DURATION OF THE BATTLE
[94]  DISENG←0.8
[95] ⍝
[96] ⍝ACTUAL ATTRITION TO BE IMPOSED ON RED FORCE
[97]  ATTR←REDDES+(DISENG×(BLUDES-REDDES))
[98] ⍝
[99] ⍝BLUE WILL STOP THE COUNTERATTACK IF BLUE RECEIVES THIS MUCH
[100] ⍝ATTRITION
[101]  BLUATT←0.5
[102] ⍝
[103] ⍝AVERAGE FIRING RATE IN ROUNDS/MINUTE
[104]  FIRRAS← 2 2 ρ 2 2 2 2
[105]  FIRRAM← 2 2 ρ 1 1 0 0
[106] ⍝
[107] ⍝PERCENTAGE OF VEHICLES CAPABLE OF FIRING
[108]  VEFIRB← 2 2 ρ 0.5 0 0.5 0.5
[109]  VEFIBR← 2 2 ρ 0.25 0.5 0 0.5
[110] ⍝
[111]  VEFIRB←VEFIRB×(⍉ 2 2 ρNUMVED[1],NUMVEI[1])
[112]  VEFIBR←VEFIBR×(2 2 ρNUMVED[2],NUMVEI[2])
[113] ⍝
[114] ⍝PERCENT OF ENGAGEMENT STATIONARY/MOVING (FIRER, TARGET)
[115]  PERSS← 2 2 ρ 0.09 0.09 0.16 0.16
[116]  PERSM← 2 2 ρ 0.81 0.01 0.64 0.64
[117]  PERMS← 2 2 ρ 0.01 0.81 0.04 0.04
[118]  PERMM← 2 2 ρ 0.09 0.09 0.16 0.16
[119] ⍝
[120] ⍝HIT PER ROUND FIRED STATIONARY/MOVING (FIRER, TARGET)
[121]  HITRSS← 2 2 ρ 0.75 0.4 0.8 0.8
[122]  HITRSM← 2 2 ρ 0.47 0.2 0.3 0.3
[123]  HITRMS← 2 2 ρ 0.25 0.1 0 0
[124]  HITRMM← 2 2 ρ 0.19 0.05 0 0
[125] ⍝
[126] ⍝HITS PER MINUTE CALCULATION 1
[127]  HTMIN1←FIRRAS×((PERSS×HITRSS)+(PERSM×HITRSM))
[128]  HTMIN1←HTMIN1+(FIRRAM×((PERMS×HITRMS)+(PERMM×HITRMM)))
[129] ⍝
[130] ⍝C3 ERROR
[131]  C3ERR← 2 2 ρ 0 0 0 0
[132] ⍝
[133] ⍝HITS PER MINUTE CALCULATION 2
[134]  HTMIN2←HTMIN1×(1-C3ERR)
[135] ⍝
[136] ⍝MAXIMUM EFFECTIVE RANGE OF WEAPON SYSTEMS IN METERS
[137]  MAXEFF← 2 2 ρ 3000 3000 15300 18100
[138] ⍝
[139] ⍝OUTPUT THE STARTING CONDITIONS
[140]  →(TIME>0)ρLOOP1
```

```
 'INITIAL START DATA'
 '   '
 'AT TIME 0 CONDITIONS ARE AS FOLLOWS:  '
 '   '
 'NUMBER OF BLUE DIRECT VEHICLES    ',(⍕NUMVED[2])
 'NUMBER OF BLUE INDIRECT VEHICLES  ',(⍕NUMVEI[2])
 'NUMBER OF RED DIRECT VEHICLES     ',(⍕NUMVED[1])
 'NUMBER OF RED INDIRECT VEHICLES   ',(⍕NUMVEI[1])
 '   '
 'RANGE BETWEEN WEAPON SYSTEMS   '
 '   '
 'BLUE DIRECT   VS.  RED DIRECT     ',(⍕RANGE[1;1])
 'BLUE DIRECT   VS.  RED INDIRECT   ',(⍕RANGE[2;1])
 'BLUE INDIRECT VS.  RED DIRECT     ',(⍕RANGE[1;2])
 'BLUE INDIRECT VS.  RED INDIRECT   ',(⍕RANGE[2;2])
 '   '
 '   '
 '   '
LOOP1:
⍝
⍝UPDATE RUN TIME OF MODEL AND CUMULATIVE TIME VECTOR
 TIME←TIME+DELTAT
 CUMTIM←CUMTIM,TIME
⍝
⍝CHANGE VEHICLE SPEED WHEN UNITS ARE ENGAGED IN DIRECT FIRE
 →(CURRAN[1;1]>3000)ρLOOP13
 VEHSPD← 2 2 ρ 10 40 0 20
⍝
LOOP13:
⍝
⍝CALCULATE THE DISTANCE EACH SYSTEM MOVES IN
⍝A TIME INTERVAL
 DSMVBD←VEHSPD[1;2]×DELTAT×(PERMS[1;2]+PERMM[1;2])
 DSMVBI←VEHSPD[2;2]×DELTAT×(PERMS[2;2]+PERMM[2;2])
 DSMVRD←VEHSPD[1;1]×DELTAT×(PERMS[1;1]+PERMM[1;1])
 DSMVRI←VEHSPD[2;1]×DELTAT×(PERMS[2;1]+PERMM[2;1])
⍝
⍝UPDATE RANGE BETWEEN OPPOSING FORCES
 CURRAN[1;1]←CURRAN[1;1]-(DSMVRD+DSMVBD)
 CURRAN[1;2]←CURRAN[1;2]-(DSMVRD+DSMVBI)
 CURRAN[2;1]←CURRAN[2;1]-(DSMVRI+DSMVBD)
 CURRAN[2;2]←CURRAN[2;2]-(DSMVRI+DSMVBI)
⍝
⍝CHECK TO INSURE THAT NO NEGATIVE RANGES OCCUR
 I←1
LOOP5:J←1
LOOP6:→(CURRAN[I;J]≥0)ρLOOP7
 CURRAN[I;J]←0
LOOP7:J←J+1
 →(J<3)ρLOOP6
 I←I+1
 →(I<3)ρLOOP5
⍝
⍝KEEP VARIABLES SEPARATE FOR RED AND BLUE RANGES
 CURRB←CURRAN
 CURBR←CURRAN
⍝
⍝UPDATE CUMULATIVE RANGE VECTOR
 CUMRAN←CUMRAN,CURRAN[1;1]
⍝
⍝IF TARGET IS OUT OF RANGE, INSURE THAT 0 CASUALTIES ARE
⍝INFLICTED
 I←1
LOOP3:J←1
LOOP2:→(CURRB[I;J]≤MAXEFF[I;1])ρLOOP4
 CURRB[I;J]←MAXEFF[I;1]
LOOP4:→(CURBR[I;J]≤MAXEFF[J;2])ρLOOP12
 CURBR[I;J]←MAXEFF[J;2]
LOOP12:J←J+1
 →(J<3)ρLOOP2
 I←I+1
 →(I<3)ρLOOP3
```

```
[213]  ⍝
[214]  ⍝RANGE VARIABLE
[215]   RRANVA← 2 2 ρ(,(1-(CURRB÷(⍉ 2 2 ρMAXEFF[;1]))))*(,⍉ 2 2 ρMU[;1])
[216]   BRANVA← 2 2 ρ(,(1-(CURBR÷(2 2 ρMAXEFF[;2]))))*(, 2 2 ρMU[;2])
[217]  ⍝
[218]  ⍝BLUE AND RED HIT PER MINUTE CALCULATION 3
[219]   RHTMN3←(⍉ 2 2 ρHTMIN2[;1])×RRANVA
[220]   BHTMN3←(2 2 ρHTMIN2[;2])×BRANVA
[221]  ⍝
[222]  ⍝FINAL ATTRITION VARIABLES FOR RED FIRER
[223]   REDID1←FIRRAS[2;1]×(PERSS[2;1]+PERSM[2;1])
[224]   REDID2←FIRRAM[2;1]×(PERMS[2;1]+PERMM[2;1])
[225]   REDID3←KPRRI[1]×(1-C3ERR[2;1])×ALLOIJ[2;1]
[226]   REDID←REDID3×(REDID1+REDID2)
[227]   REDII←KPRRI[2]×(1-C3ERR[2;1])×(REDID1+REDID2)×ALLOIJ[2;2]
[228]   AIJ←RHTMN3×KILLHR×ALLOIJ
[229]   AIJ← 2 2 ρ(AIJ[1;1],AIJ[1;2],REDID,REDII)
[230]  ⍝
[231]  ⍝FINAL ATTRITION VARIABLES FOR BLUE FIRER
[232]   BLUID1←FIRRAS[2;2]×(PERSS[2;2]+PERSM[2;2])
[233]   BLUID2←FIRRAM[2;2]×(PERMS[2;2]+PERMM[2;2])
[234]   BLUID3←KPRBI[1]×(1-C3ERR[2;2])×ALLOJI[1;2]
[235]   BLUID←BLUID3×(BLUID2+BLUID1)
[236]   BLUII←KPRBI[2]×ALLOJI[2;2]×(1-C3ERR[2;2])×(BLUID1+BLUID2)
[237]   AJI←BHTMN3×KILLHB×ALLOJI
[238]   AJI← 2 2 ρ(AJI[1;1],BLUID,AJI[2;1],BLUII)
[239]  ⍝
[240]  ⍝MISCELLANEOUS LETHALITY AND VULNERABILITY MULTIPLIERS
[241]   MISCR←MISCLE×MISCVU
[242]   MISCB←MISCLE×MISCVU
[243]   MISCR←⍉ 2 2 ρMISCR[;1]
[244]   MISCB← 2 2 ρMISCB[;2]
[245]  ⍝
[246]  ⍝UPDATE ATTRITION MATRIX
[247]   AIJ←AIJ×MISCR
[248]   AJI←AJI×MISCB
[249]  ⍝
[250]  ⍝CASUALTIES CALCULATED BY FIRER AT TARGET
[251]   CARDBD←AIJ[1;1]×VEFIRB[1;1]×DELTAT
[252]   CARIBD←AIJ[2;1]×VEFIRB[2;1]×NUMVED[2]×DELTAT
[253]   CARIBI←AIJ[2;2]×NUMVEI[2]×VEFIRB[2;2]×DELTAT
[254]   CABDRD←AJI[1;1]×VEFIBR[1;1]×DELTAT
[255]   CABIRD←AJI[1;2]×NUMVED[1]×VEFIBR[1;2]×DELTAT
[256]   CABIRI←AJI[2;2]×NUMVEI[1]×VEFIBR[2;2]×DELTAT
[257]   CASBI←CARIBI
[258]   CASRI←CABIRI
[259]   CASBD←CARDBD+CARIBD
[260]   CASRD←CABDRD+CABIRD
[261]  ⍝
[262]  ⍝TOTAL CASUALTIES
[263]   CASD←(CASRD,CASBD)
[264]   CASI←(CASRI,CASBI)
[265]  ⍝
[266]  ⍝UPDATE FORCE SIZE NUMBER OF DIRECT AND INDIRECT VEHICLES
[267]   NUMVED←(NUMVED-CASD)⌈0.001
[268]   NUMVEI←(NUMVEI-CASI)⌈0.001
[269]  ⍝
[270]  ⍝UPDATE CUMULATIVE LOSSES BY FIRER PER TARGET
[271]   LOBDRD←LOBDRD,CABDRD
[272]   LOBIRD←LOBIRD,CABIRD
[273]   LOBIRI←LOBIRI,CABIRI
[274]   LORDBD←LORDBD,CARDBD
[275]   LORIBD←LORIBD,CARIBD
[276]   LORIBI←LORIBI,CARIBI
[277]  ⍝
[278]  ⍝PERCENT OF VEHICLES SURVIVING BY VEHICLE TYPE
[279]  ⍝
[280]   BLUDIR←NUMVED[2]÷INITLD[2]
[281]   BLUIND←(NUMVEI[2]÷INITLI[2])⌈0
[282]   REDDIR←NUMVED[1]÷INITLD[1]
[283]   REDIND←(NUMVEI[1]÷INITLI[1])⌈0
[284]   BIRATI←BIRATI,(NUMVEI[2]÷NUMVED[2])
```

```
⍝
 →(REDDIR>(1-ATTR))⍴LOOP22
 →(CRD=1)⍴LOOP40
 'REDDIR AFFECTED'
 CRD←1
LOOP40:
 ALLOIJ[1;1]←0
 ALLOJI← 2 2 ⍴ 0 0 0 1
LOOP22:
 →(BLUDIR>BLUATT)⍴LOOP23
 →(CBD=1)⍴LOOP41
 'BLUDIR AFFECTED'
 CBD←1
LOOP41:
 ALLOJI[1;1]←0
 ALLOIJ← 2 2 ⍴ 0 0 0 1
LOOP23:
 →(BLUIND>0.3)⍴LOOP24
 →(CBI=1)⍴LOOP42
 'BLUIND AFFECTED'
 CBI←1
LOOP42:
 ALLOJI← 2 2 ⍴ 1 0 0 0
 ALLOIJ← 2 2 ⍴ 1 0 1 0
LOOP24:
 →(REDIND>0.3)⍴LOOP25
 →(CRI=1)⍴LOOP43
 'REDIND AFFECTED'
 CRI←1
LOOP43:
 ALLOJI← 2 2 ⍴ 1 1 0 0
 ALLOIJ← 2 2 ⍴ 1 0 0 0
LOOP25:
 →(((REDDIR<(1-ATTR))∧(BLUDIR<BLUATT))=0)⍴LOOP26
 →(CRDBD=1)⍴LOOP44
 'BOTH RD AND BD AFFECTED'
 CRDBD←1
LOOP44:
 ALLOIJ← 2 2 ⍴ 0 0 0 1
 ALLOJI← 2 2 ⍴ 0 0 0 1
LOOP26:
 →(((REDIND<0.3)∧(BLUIND<0.3))=0)⍴LOOP27
 →(CRIBI=1)⍴LOOP45
 'BOTH RI AND BI AFFECTED'
 CRIBI←1
LOOP45:
 ALLOIJ[2;]←0
 ALLOJI[;2]←0
LOOP27:
 →(((REDDIR<(1-ATTR))∧(BLUIND<0.3))=0)⍴LOOP28
 →(CRDBI=1)⍴LOOP46
 'BOTH RD AND BI AFFECTED'
 CRDBI←1
LOOP46:
 ALLOIJ← 2 2 ⍴ 0 0 1 0
 ALLOJI← 2 2 ⍴ 0 0 0 0
LOOP28:
 →(((REDIND<0.3)∧(BLUDIR<BLUATT))=0)⍴LOOP29
 →(CRIBD=1)⍴LOOP47
 'BOTH RI AND BD AFFECTED'
 CRIBD←1
LOOP47:
 ALLOIJ← 2 2 ⍴ 0 0 0 0
 ALLOJI← 2 2 ⍴ 0 1 0 0
LOOP29:
 TOTAL←(+/LORDBD),(+/LORIBD),(+/LORIBI),(+/LOBDRD),(+/LOBIRD),+/LOBIRI
⍝OUTPUT DESIRED TIME INTERVALS
 →((TIME=1)∨((1-REDDIR)≥ATTR)∨((TIME÷10)=(⌊(TIME÷10))))⍴LOOP20
 →(((1-BLUDIR)≥BLUATT)∨((|(CURRAN[1;1]-3000))≤50))⍴LOOP20
⍝→LOOP21
⍝
LOOP20:
```

```
⍝
 'TIME IS     ',(⍕TIME),'  RANGE IS    ',(⍕CURRAN[1;1])
⍝
LOOP21:

⍝IF BREAKPOINT CRITERIA HAS BEEN REACHED STOP BATTLE
 →((BLUIND<0.3)∧(BLUDIR<BLUATT))ρLOOP8
 →((REDIND<0.3)∧(REDDIR<(1-ATTR)))ρLOOP9
⍝
⍝TIME STOPPING CRITERIA
 →(TIME<100)ρLOOP1
 →LOOP10
⍝
LOOP8:
 '    '
 'MISSION FAILED   '
 '   '
 '   '
 'BLUE RECEIVED     ',(⍕(1-BLUDIR)),'  PERCENT CASUALTIES'
⍝
 COUNT←COUNT+1
 RD←RD,(1-REDDIR)
 BD←BD,(1-BLUDIR)
 RI←RI,(1-REDIND)
 BI←BI,(1-BLUIND)
⍝→(INITLI[2]≤500)ρLOOP80
 →LOOP11
⍝
LOOP9:
 '    '
 '    '
 'MISSION SUCCESSFUL!!!'
 '  '
 'RED RECEIVED      ',(⍕(1-REDDIR)),'  PERCENT CASUALTIES'
⍝
 COUNT←COUNT+1
 RD←RD,(1-REDDIR)
 BD←BD,(1-BLUDIR)
 RI←RI,(1-REDIND)
 BI←BI,(1-BLUIND)
⍝→(INITLI[2]≤500)ρLOOP80
 →LOOP11
LOOP10:
 'TIME EXPIRED'
 '  '
 COUNT←COUNT+1
 RD←RD,(1-REDDIR)
 BD←BD,(1-BLUDIR)
 RI←RI,(1-REDIND)
 BI←BI,(1-BLUIND)
⍝→(INITLI[2]≤500)ρLOOP80
LOOP11:
 '  '
 'FINAL BLUE DIRECT FIRE CASUALTIES   ',(⍕(1-BLUDIR))
 '   '
 'FINAL RED  DIRECT FIRE CASUALTIES   ',(⍕(1-REDDIR))
 '   '
 '   '
 'RESULTING ATTRITION MATRICES ARE:'
 'AIJ'
 '   '
 ⍕AIJ
 '  '
 '  '
 'AJI   '
 '    '
 ⍕AJI
⍝
 '    '
 '    '
 ' FINAL BATTLE OUTPUT '
 '  '
```

```
[429]  '   FIRER                  TARGET           NUMBER KILLED'
[430]  '  '
[431]  'RED     DIRECT    BLUE    DIRECT       ',(⍕+/LORDBD)
[432]  'RED   INDIRECT    BLUE    DIRECT       ',(⍕+/LORIBD)
[433]  'RED   INDIRECT    BLUE  INDIRECT       ',(⍕+/LORIBI)
[434]  '  '
[435]  'BLUE    DIRECT    RED     DIRECT       ',(⍕+/LOBDRD)
[436]  'BLUE  INDIRECT    RED     DIRECT       ',(⍕+/LOBIRD)
[437]  'BLUE  INDIRECT    RED   INDIRECT       ',(⍕+/LOBIRI)
```

A SAMPLE OUTPUT IS LISTED BELOW.

```
INPUT DESIRED NUMBER OF DIRECT VEHICLES
⎕:
      6228
INPUT DESIRED NUMBER OF INDIRECT VEHICLES
⎕:
      484
INITIAL START DATA

AT TIME 0 CONDITIONS ARE AS FOLLOWS:

NUMBER OF BLUE DIRECT VEHICLES      6228
NUMBER OF BLUE INDIRECT VEHICLES    484
NUMBER OF RED DIRECT VEHICLES       1557
NUMBER OF RED INDIRECT VEHICLES     424

RANGE BETWEEN WEAPON SYSTEMS

BLUE DIRECT   VS.  RED DIRECT      5000
BLUE DIRECT   VS.  RED INDIRECT    10000
BLUE INDIRECT VS.  RED DIRECT      10000
BLUE INDIRECT VS.  RED INDIRECT    15000


TIME IS      1   RANGE IS     4886
TIME IS      2   RANGE IS     4772
TIME IS      3   RANGE IS     4658
TIME IS      4   RANGE IS     4544
TIME IS      5   RANGE IS     4430
TIME IS      6   RANGE IS     4316
TIME IS      7   RANGE IS     4202
TIME IS      8   RANGE IS     4088
TIME IS      9   RANGE IS     3974
TIME IS      10   RANGE IS     3860
TIME IS      11   RANGE IS     3746
TIME IS      12   RANGE IS     3632
TIME IS      13   RANGE IS     3518
TIME IS      14   RANGE IS     3404
TIME IS      15   RANGE IS     3290
TIME IS      16   RANGE IS     3176
TIME IS      17   RANGE IS     3062
TIME IS      18   RANGE IS     2948
TIME IS      19   RANGE IS     2911
TIME IS      20   RANGE IS     2874
TIME IS      21   RANGE IS     2837
TIME IS      22   RANGE IS     2800
TIME IS      23   RANGE IS     2763
TIME IS      24   RANGE IS     2726
TIME IS      25   RANGE IS     2689
TIME IS      26   RANGE IS     2652
TIME IS      27   RANGE IS     2615
TIME IS      28   RANGE IS     2578
TIME IS      29   RANGE IS     2541
TIME IS      30   RANGE IS     2504
TIME IS      31   RANGE IS     2467
TIME IS      32   RANGE IS     2430
TIME IS      33   RANGE IS     2393
TIME IS      34   RANGE IS     2356
TIME IS      35   RANGE IS     2319
TIME IS      36   RANGE IS     2282
TIME IS      37  RANGE IS     2245
TIME IS      38  RANGE IS     2208
TIME IS      39  RANGE IS     2171
TIME IS      40  RANGE IS     2134
TIME IS      41  RANGE IS     2097
BLUDIR AFFECTED
TIME IS      42  RANGE IS     2060
TIME IS      43  RANGE IS     2023
TIME IS      44  RANGE IS     1986
TIME IS      45  RANGE IS     1949
TIME IS      46  RANGE IS     1912
REDDIR AFFECTED
BOTH RD AND BD AFFECTED
TIME IS      47  RANGE IS     1875
TIME IS      48  RANGE IS     1838
TIME IS      49  RANGE IS     1801
TIME IS      50  RANGE IS     1764
TIME IS      51  RANGE IS     1727
TIME IS      52  RANGE IS     1690
TIME IS      53  RANGE IS     1653
TIME IS      54  RANGE IS     1616
TIME IS      55  RANGE IS     1579
TIME IS      56  RANGE IS     1542
TIME IS      57  RANGE IS     1505
TIME IS      58  RANGE IS     1468
REDIND AFFECTED
BOTH RI AND BD AFFECTED
TIME IS      59  RANGE IS     1431

MISSION SUCCESSFUL!!!

RED RECEIVED       0.7014500412  PERCENT CASUALTIES

FINAL BLUE DIRECT FIRE CASUALTIES    0.5081024927

FINAL RED  DIRECT FIRE CASUALTIES    0.7014500412


RESULTING ATTRITION MATRICES ARE:
AIJ

 0        0
 0        0.000096


AJI

 0        0
 0        0.000192


 FINAL BATTLE OUTPUT

   FIRER              TARGET           NUMBER KILLED

RED     DIRECT     BLUE    DIRECT     2259.454865
RED   INDIRECT     BLUE    DIRECT     905.0074596
RED   INDIRECT     BLUE  INDIRECT     241.9319855

BLUE    DIRECT     RED     DIRECT     796.7291719
BLUE  INDIRECT     RED     DIRECT     295.4285422
BLUE  INDIRECT     RED   INDIRECT     300.153251
```

APPENDIX B. INPUTS USED IN THE MODEL

The following list shows the current values and shape of the variables that are used in the APL program (shape corresponds to the size of the vector or matrix). Abbreviations used in the application column (i.e. BD represents Blue direct weapon systems) specify the location within the vector or matrix that applies to the weapon system type.

VARIABLE NAME	VALUES		APPLICATION		SHAPE	
ALLOIJ	1.00	0.00	RD-BD	RD-BI	2	2
	.60	.40	RI-BD	RI-BI		
ALLOJI	1.00	.70	BD-RD	BI-RD	2	2
	.00	.30	BD-RI	BI-RI		
BLUATT	.50				0	
BLUDES	.75				0	
C3EPP	.00	.00	RD	BD	2	2
	.00	.00	RI	BI		
DELTAT	1.00				0	
DISENG	.80				0	
FIRRAM	1.00	1.00	RD	BD	2	2
	0.00	0.00	RI	BI		
FIRRAS	2.00	2.00	RD	BD	2	2
	2.00	2.00	RI	BI		
HITRMM	.19	.05	RD	BD	2	2
	0.00	0.00	RI	BI		

```
HITRMS               .25        .10     RD       BD          2  2
                    0.00       0.00     RI       BI

HITRSM               .47        .20     RD       BD          2  2
                     .30        .30     RI       BI

HITRSS               .75        .40     RD       BD          2  2
                     .80        .80     RI       BI

KILLHB               .65        .10     BD-RD    BI-RD       2  2
                    0.00        .40     BD-RI    BI-RI

KILLHR               .65       0.00     RD-BD    RD-BI       2  2
                     .10        .40     RI-BD    RI-BI

KPRBI                .00002     .00012  BI-RD    BI-RI       2

KPRRI                .00002     .00006  RI-BD    RI-BI       2

MAXEFF           3000.00    3000.00     RD       BD          2  2
                15300.00   18100.00     RI       BI

MISCLE              1.00       1.00     RD       BD          2  2
                    1.00       1.00     RI       BI

MISCVU              1.00       1.00     RD       BD          2  2
                    1.00       1.00     RI       BI

MU                   .90        .90     RD       BD          2  2
                    0.00       0.00     RI       BI

NUMVED           1557.00    2976.00     RD       BD          2

NUMVEI            424.00     243.00     RI       BI          2

PERMM                .09        .09     RD       BD          2  2
                     .16        .16     RI       BI

PERMS                .01        .81     RD       BD          2  2
                     .04        .04     RI       BI

PERSM                .81        .01     RD       BD          2  2
                     .64        .64     RI       BI

PERSS                .09        .09     RD       BD          2  2
                     .16        .16     RI       BI

RANGE            5000.00   10000.00     RD-BD    RD-BI       2  2
                10000.00   15000.00     RI-BD    RI-BI
```

```
REDDES          .50                                   0

VEFIBR          .25      .50    BD-RD    BI-RD        2  2
                .00      .50    BD-RI    BI-RI

VEFIRB          .50     0.00    RD-BD    RD-BI        2  2
                .50      .50    RI-BD    RI-BI

VEHSPD        60.00   120.00    RD       BD           2  2
              20.00    50.00    RI       BI
```

APPENDIX C. ATTRITION COEFFICIENT CALCULATIONS

The formulas used to calculate the attrition coefficients by weapon system (A_{ij}) for use with the Lanchester attrition equations are shown below. More detailed explanations on Lanchesterian attrition theory and calculations is given in Taylor [Ref. 6].

For direct fire weapons:

$$A_{ij} = P_{ij} x HT_{ijk} x (\%\mathrm{M} x v_{im} + \%S_i x v_{is}) x (1 - C3_i) x \left(1 - \frac{R}{R_i}\right)^{\mu_i} x \Psi_{ij} x \mathrm{MLE} x \mathrm{MVU} \quad (16)$$

having units of (# killed)/(# firers)(unit time).

For indirect fire weapons:

$$A_{ij} = Q_{ij} x (\%\mathrm{M} x v_{im} + \%S_i x v_{is}) x (1 - C3_i) x \Psi_{ij} x \mathrm{MLE} x \mathrm{MVU} \quad (17)$$

having units of (# killed)/(# firers)(# targets)(unit time).

Where,

A_{ij} = Attrition coefficient for i firer, j target

P_{ij} =Probability of kill given hit (i firer, j target)

Q_{ij} =Probability of kill given shot (i firer, j target)

HT_{ijk} =Probability of hit per shot (i firer, j target, k moving or stationary)

$\%M_i$ =Percent of the time system i is moving

v_{im} =System i rate of fire when moving

$\%S_i$ =Percent of the time system i is stationary

v_{is} =System i rate of fire when stationary

$C3_i$ =Degradation due to intelligence error for system i.

R =Current distance between forces.

R_i =Maximum effective range of system i.

u_i =Weapon accuracy parameter for system i.

Y_{ij} =Percent of allocation of i firer to j target.

MLE =Miscellaneous lethality multiplier.

MVU =Miscellaneous vulnerability multiplier.

Therefore, using the convention of X as the Blue force and Y as the Red force, with a and b as the attrition coefficients (from Equations 16 and 17) for the Blue and Red forces, respectively, the following equations are produced.

For Blue direct attrition,

$$dX_D/dt = -b_2X_DY_I \quad -b_1Y_D \tag{18}$$

and for Blue indirect attrition,

$$dX_I/dt = -b_3X_IY_I \tag{19}$$

Similarly, the direct attrition of Red becomes:

$$dY_D/dt = -a_2Y_DX_I \quad -a_1X_D \tag{20}$$

and for Red indirect systems,

$$dY_Idt = -a_3Y_IX_I \tag{21}$$

These equations are described in Table 3 of Chapter 3.

APPENDIX D. RELATED GRAPHS

The following charts (Figures 11 and 12) show another method of representing separate attrition effects during an engagement. Recall from Chapter 4 that the Red threat consisted of five distinct scenarios. Only one scenario (Scenario 5) is shown in the following graphs.

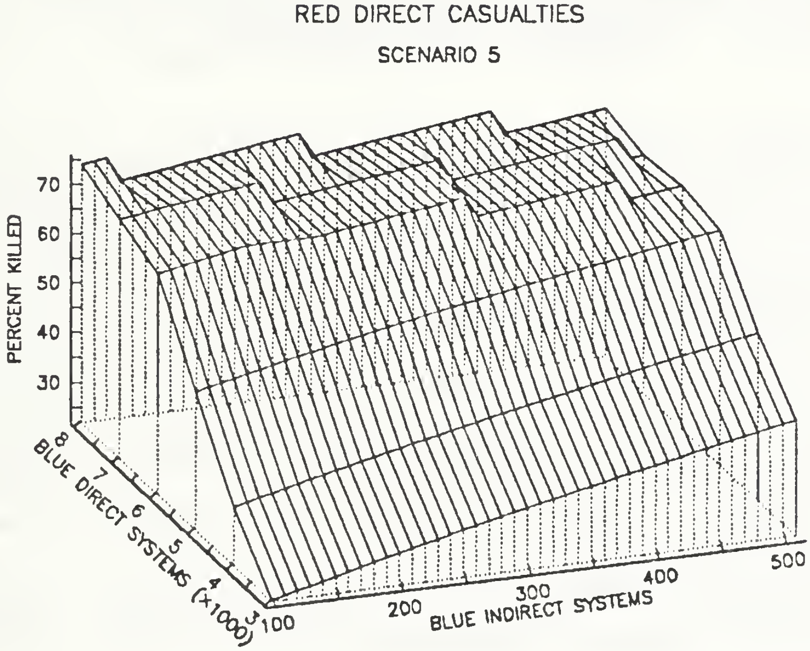

Figure 11. 3-Dimensional Graph depicting Red direct attrition

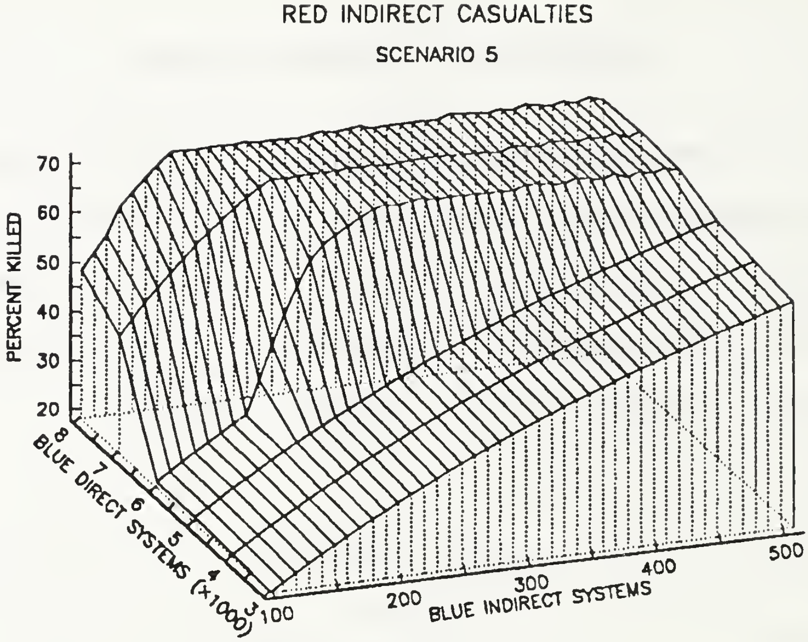

Figure 12. 3-Dimensional Graph depicting Red indirect attrition

Surface graphs (i.e. Figures 11 and 12) render a broader view of the heterogeneous model's performance. Recall the analysis discussion in Chapter 4 reference Scenario 5. The Red threat was characterized by possessing 500 indirect weapon systems. Analysis from these graphs allow planar cuts along the X and Y axis to represent the four cases of Blue force structures discussed in detail in Chapter 4.

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